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Vortex-induced vibration analysis of long-span bridge in time domain by finite elements

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SUMMARY:
With the development of computer technology, the study of vortex-induced vibration analysis by finite element method is gradually deepened. In this paper, the existing two-dimensional vortex-induced force model is derived into the three-dimensional finite element vortex-induced force model, then a calculation method of vortex-induced vibration response based on segmental model wind tunnel test and existing vortex-induced force model is proposed by mathematic software MATLAB. The vortex-induced vibration responses of a simply supported beam bridge and a long-span cable-stayed bridge are calculated respectively. The accuracy of the calculation method in the present study is verified by the results of simply supported beam bridge. Additionally, the vortex-induced vibration amplitude of complex long-span bridge is larger than that of simply supported beam bridge, which shows that the results obtained by traditional estimation methods will underestimate the amplitude of vortex-induced vibration. This calculation method can provide a reference for studying vortex-induced vibration response of long-span bridges.

Keywords: Long-span bridge, wind tunnel test, vortex-induced vibration

1. INSTRUCTION

When the wind flows across a bluff body, it will produce periodic shedding vortices, which will cause vortex-induced vibration (VIV) of the structure. At present, there are many research methods about VIV, but due to the complexity of VIV, the semi-empirical vortex-induced force (VIF) model methods based on wind tunnel test results are widely used (Ehsan and Scanlan, 1990; Larsen, 1995; Simiu and Scanlan, 1996). However, the results of segmental wind tunnel test are two-dimensional, which cannot reflect the effect of mode shape and three-dimensional of the actual bridge on the VIV response. For several common semi-empirical VIF models, the effect of simple mode shape on VIV amplitude is studied by Ge et al. (2014). The impact factors of different VIF models from the sectional model to the actual bridge are given, but the impact factors are not accurate for complex modes shape. The *Newmark-β* method to simulate the VIV in time domain can automatically consider the effect of all mode shapes, and can give the whole VIV time history vividly by finite elements. In this paper, based on the widely used Scanlan semi-empirical nonlinear VIF model and sectional model wind tunnel test, the VIV time history response of a simply supported beam bridge and a long-span cable-stayed bridges are calculated in time domain by mathematic software MATLAB.

2. VORTEX-INDUCED FORCE MODELS

2.1. Scanlan semi-empirical nonlinear model

In practical engineering, designers are usually concerned about the amplitude of the continuous VIV to evaluate the degree of fatigue damage to the bridge. To establish a simple and practical VIF model to meet the engineering needs, Simiu and Scanlan (1996) used the mechanical oscillator model to describe the VIV problem, and proposed a semi-empirical linear model, which can obtain the amplitude of VIV by analytical solution. However, VIV is a kind of nonlinear wind-induced vibration, and the linear model can only describe the VIV process in the lock-in stage, which cannot explain the nonlinear phenomenon in VIV. Therefore, based on the semi-empirical linear model, Scanlan introduced a nonlinear aerodynamic term, which can be expressed as:

$$\eta''+2\xi K\eta'+K^2\eta = \rho D^2/m\left[Y_1(K)(1-\varepsilon\eta^2)\eta'+Y_2(K)\eta+\frac{1}{2}C_L(K)\sin(Ks+\phi)\right] \quad (1)$$

Where m is the mass per unit length of the model, ω_0 is the structural vibration frequency, D is the characteristic size of the structure, η is dimensionless amplitude, $\eta=y/D$, ρ is air density, ζ is the damping ratio of the system, U is wind speed, s is dimensionless time, $s=Ut/D$, K is dimensionless reduced frequency, $K=\omega_s D/U$, ω_s is vortex shedding frequency, Y_1 (K), ε, Y_2 (K), $C_L(K)$ are aerodynamic parameters to be identified.

The dimensionless stable VIV amplitude is

$$\eta_\infty = 2\sqrt{\frac{1}{\varepsilon}-\frac{2\xi K}{\rho D^2/mY_1(K)\varepsilon}} \quad (2)$$

2.1. Finite element VIF model

Most of the current VIF models are based on the results of sectional model wind tunnel tests. Thus, it is necessary to extend these models to finite element models. Barhoush et al. (1995) deduced the existing VIF model to finite element VIF model and verified it with a simple example. In this section, using the above method, the Scanlan semi-empirical nonlinear VIF model is derived to finite element VIF model. According to the principle of virtual displacement, in the time of t_1 ~ t_2, the work done by all forces is 0.

$$\int_{t_1}^{t_2}\delta(T-U+W)\mathrm{d}t = 0 \quad (3)$$

Where δ is the variational operator in a specified time interval, T denotes the total kinetic energy of the system, U is the total potential energy of the system, W is the virtual work of non-conservative force acting on the system. Substituting the work done by each force into Eq. (3).

$$\int_{t_1}^{t_2}\int_0^L\left[\begin{array}{l}-\rho A\ddot{v}\delta v-EIv''\delta v''-c\dot{v}\delta v+\frac{1}{2}\rho U^2(2D)\\ \times\left(Y_1(K)\left(1-\varepsilon\frac{v^2}{D^2}\right)\frac{\dot{v}}{U}+Y_2(K)\frac{v}{D}+\frac{1}{2}C_L(K)\sin(\omega t+\phi)\right)\delta v\end{array}\right]\mathrm{d}x\mathrm{d}t = 0 \quad (4)$$

Where L is the element length, A is the cross-sectional area of the unit, c is the damping ratio of the structure, $v(x, t)$ is the node displacement. The node displacement can be expressed as the shape function $\Psi(x)$ and the displacement of a single node $v(t)$.

$$v(x,t)=\{\psi(x)\}^{\mathrm{T}}\{v(t)\} \quad (5)$$

Thus, the Eq. (4) can be rewritten as:

$$\int_{t_1}^{t_2}\left(-\{\delta v\}^{\mathrm{T}}[M]\left\{\ddot{v}\right\}-\{\delta v\}^{\mathrm{T}}[K]\{v\}-\{\delta v\}^{\mathrm{T}}[C]\left\{\dot{v}\right\}-\{\delta v\}^{\mathrm{T}}[D_{\mathrm{VIF}}]\left\{\dot{v}\right\}\right)=0 \tag{6}$$

$$[K]=\int_0^L EI\{\psi''\}\{\psi''\}^{\mathrm{T}}\,\mathrm{d}x \qquad [M]=\int_0^L \rho A\{\psi\}\{\psi\}^{\mathrm{T}}\,\mathrm{d}x \tag{7}$$

$$[C]=\int_0^L c_{ij}\{\psi\}\{\psi\}^{\mathrm{T}}\,\mathrm{d}x,\ i=1,\cdots,4,\ \ j=1,\cdots 4 \tag{8}$$

Where D_{VIF} is the work done by VIF.

Since the vortex shedding force is small relative to the self-excited force in the VIV lock-in stage, C_L (K) can be ignored (Ehsan and Scanlan, 1990) and the vortex shedding frequency in the lock-in stage is locked by the model vibration frequency, so $Y_2(K)$ can also be ignored (Simiu and Scanlan, 1996). Finally, only the linear term D_L and the nonlinear term D_N are left.

$$[D_L]=\frac{1}{2}\rho U^2(2D)\frac{Y_1(K)}{U}\int_0^L\{\psi\}\{\psi\}^{\mathrm{T}}\,\mathrm{d}x=\alpha_1\int_0^L\{\psi\}\{\psi\}^{\mathrm{T}}\,\mathrm{d}x \tag{9}$$

$$\alpha_1=\frac{1}{2}\rho U(2D)Y_1(K) \qquad \alpha_2=\frac{1}{2}\rho U(2D)Y_1(K)\left(\varepsilon/D^2\right) \tag{10}$$

$$\begin{aligned}[D_N]&=\frac{1}{2}\rho U^2(2D)\varepsilon\frac{Y_1(K)}{D^2U}\int_0^L(\psi_1 v_1+\psi_2 v_2+\psi_3 v_3+\psi_4 v_4)\{\psi\}\{\psi\}^{\mathrm{T}}\,\mathrm{d}x\\&=\alpha_2\int_0^L(\psi_1 v_1+\psi_2 v_2+\psi_3 v_3+\psi_4 v_4)\{\psi\}\{\psi\}^{\mathrm{T}}\,\mathrm{d}x\end{aligned} \tag{11}$$

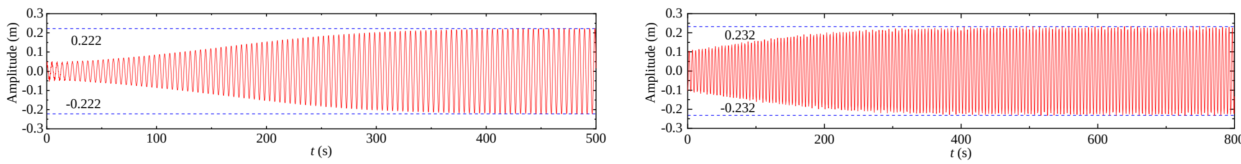
Therefore, the Eq. (6) can be simplified as:

$$[M]^e\left\{\ddot{v}\right\}+[C]^e\left\{\dot{v}\right\}+[K]^e\{v\}=\alpha_1[D_L]^e\left\{\dot{v}\right\}-\alpha_2[D_N]^e\left\{\dot{v}\right\} \tag{12}$$

3. VIV ANALYSIS IN TIME DOMAIN

A long-span cable-stayed bridge is taken as the engineering background in the present study. The main span of the bridge is 938 m, the both side spans are 350 m. The width and height of main girder are 48 m and 4.5 m. The scale ratio of the sectional model is 1: 50, thus the size of the model is 0.09 m (height)×0.96 m (width)×2.095 m (length). In addition, the mass of the sectional model is 18.98 kg/m and the damping ratio is 0.35%. The vertical vibration frequencies of the actual bridge and the sectional model are 0.215 Hz and 2.39 Hz respectively. Through the wind tunnel test, it is found that the VIV amplitude of the actual bridge reaches to 0.192 m at +3° attack angle and 20.5 m/s wind velocity, the corresponding dimensionless amplitude is 0.0427. Using the parameter identification method proposed by Ehsan and Scanlan (1990) in the Scanlan semi-empirical nonlinear VIF model, the parameters $Y_1(K)$ and ε are identified as 8.8145 and 1372.3, and the theoretical dimensionless stable amplitude is 0.0431 from Eq. (2). The error between the experimental value and the theoretical value is very small, indicating that the identified parameters are accurate.

To verify the accuracy of the calculation, a simply supported beam bridge is first used for verification. The parameters such as mass, damping and stiffness of the simply supported beam bridge are the same as those of the actual bridge. The results are shown in Figure 1a. Since the VIF contains a self-excited component associated with motion, an initial displacement of the system needs to be given to begin the calculation, so the initial amplitude does not start from 0. As the time t increases, the amplitude of the mid-span gradually increases. When it increases to about 0.222 m, the amplitude no longer increases, and the vibration continues with a stable amplitude. The stable amplitude is about $2/\sqrt{3}$ times larger than the results of sectional model wind tunnel test, which is the same as the results of traditional method (multiplying impact factors) proposed by Ge et al. (2014). It is shown that the amplitude after considering the mode shape will be larger than that without considering the mode shape. Figure 1b presents the mid-span displacement time history of actual bridge. Obviously, the stable VIV amplitude in Figure 1b is about 0.232 m and larger than that of simply supported beam bridge. This is because the mode shape and boundary conditions of actual bridge is much more complex than those of simply supported beam bridge. Therefore, for complex long-span bridges, it is more dangerous to estimate the amplitude of actual bridge by traditional method, which will underestimate the influence of VIV on the actual bridges.



(a) Simply supported beam bridge (b) Actual bridge

Figure 1. The displacement time history of the mid-span

4. CONCLUSIONS

The VIV responses of a simply supported beam bridge and a long-span cable-stayed bridge are calculated by the calculation method proposed in the present study. The VIV response results of simply supported girder bridge are consistent with the results of traditional method, which shows this calculation method has high accuracy. In addition, the VIV amplitude at mid-span of long-span bridge is larger than the results of simply supported beam bridge, indicating the traditional method will underestimate the VIV amplitude of the actual bridge. This calculation method can be extended to other VIV analysis of complex long-span bridges and can also be based on other VIF models.

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